Universidad
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Physics<br>Biomedical Engineering<br>Course 2020-2021

## Problem Sheet 2

Kinematics

1. The motion of a particle in $X Y$ plane is $\mathbf{r}(\mathbf{t})=(1+\cos 2 \mathbf{t}, 2+\sin 2 t)$, SI units. a) Find the velocity $\mathbf{v}(\mathbf{t})$ and the acceleration $\mathbf{a}(\mathbf{t})$. b) Find the value for $\mathbf{v}(\mathbf{1})$ and $\mathbf{a}(\mathbf{1})$. c) Find the intrinsic components of the acceleration, $\overrightarrow{a_{t}}(t)$ and $\overrightarrow{a_{n}}(t)$. d) Determine the trajectory of the particle.
Ans: a) $\left.\mathrm{v}(\mathrm{t})=(-2 \sin 2 \mathrm{t}, 2 \cos 2 \mathrm{t}), \mathrm{a}(\mathrm{t})=(-4 \cos 2 \mathrm{t},-4 \sin 2 \mathrm{t}) ; \mathrm{b}) \vec{v}(1)=(-1,819,-0,832) \mathrm{m} / \mathrm{s} ; \vec{a}(1)=(1,67 ; 3,64) \mathrm{m} / \mathrm{s}^{2} ; \mathrm{c}\right) \overrightarrow{a_{t}}(t)=0 ; \overrightarrow{a_{n}}(t)=\vec{a}(t)$ $\left(\mathrm{m} / \mathrm{s}^{2}\right) ;(x-1)^{2}+(y-2)^{2}=1 \equiv \mathrm{~A}$ circumference with radius 1 m and center in point $(1,2)$
2. The components of the position vector of a body moving in the XY plane are: $x=16 t+2 t^{2}, y=18 t-1,5 t^{2}$, SI units. At $t=1 \mathrm{~s}$, calculate: a) Cartesian coordinates and intrinsic components of the acceleration. b) The unit vectors $\mathbf{u}_{\mathbf{t}}$ and $\mathbf{u}_{\mathbf{n}}$, showing that they are perpendicular to each other. c) The position of the center of curvature. Ans.: a) $\left.\mathrm{a}_{\mathrm{x}}=4 \mathrm{~m} / \mathrm{s}^{2} ; \mathrm{a}_{\mathrm{y}}=-3 \mathrm{~m} / \mathrm{s}^{2} ; \mathrm{a}_{\mathrm{t}}=1,4 \mathrm{~m} / \mathrm{s}^{2} ; \mathrm{a}_{\mathrm{n}}=4,8 \mathrm{~m} / \mathrm{s}^{2} ; \mathrm{b}\right) \mathrm{u}_{\mathrm{t}}=(0,8,0,6) ; \mathrm{u}_{\mathrm{n}}=(0,6,-0,8) ;$ c) C.C: $\mathrm{x}=96,12 \mathrm{~m}, \mathrm{y}=-87,67 \mathrm{~m}$
3. A particle is moving in XY plane with $a_{x}=0, a_{y}=5 \mathrm{~m} / \mathrm{s}^{2}$. At $t=0$ the position is $x=0, y=8 \mathrm{~m}$, and the velocity is $v_{x}=5, v_{y}=0 \mathrm{~m} / \mathrm{s}$. Determine: a) The law of motion $\left.\vec{r}(t) ; \mathrm{b}\right)$ the equation of the trajectory; c) the intrinsic components of the acceleration at $t=1 \mathrm{~s}$. Ans.: $r(t)=\left(5 t, 8+5 / 2 \cdot t^{2}\right) ; y=8+x^{2} / 10$; $a_{t}(1)=5 / \sqrt{2} m s^{-2} ; a_{n}(1)=5 / \sqrt{2} \mathrm{~ms}^{-2}$.
4. An iron sphere is dropped and allowed to fall freely from a balcony. It takes to it $0,10 \mathrm{~s}$ for going through the last meter. a) What is the initial height? b) What is the final speed? Ans.: a) $5,5 \mathrm{~m}$; b) $10,5 \mathrm{~m} / \mathrm{s}$
5. A stone is thrown vertically upwards and is 10 s in the air. Neglecting the air resistance and taking $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$, calculate the initial throwing speed and the maximum height the stone reaches. Ans.: $50 \mathrm{~m} / \mathrm{s} ; 125 \mathrm{~m}$.
6. A child throws horizontally a ball from a balcony at height $y_{0}$, with an initial speed $\mathrm{v}_{0}$. At the same instant, another child on the street throws another ball vertically with speed $2 \mathrm{v}_{0}$, from a point located at distance $d$ from the first building. Find the value of $d$ for making the balls to collide to each other. Ans.: $d=y_{0} / 2$
7. When a stone is thrown with speed $\mathrm{v}_{0}$ at an angle $\alpha$, it hits the ground at a distance of 50 m (neglecting the air resistance). If the same stone is thrown at the same angle but with speed $2 \mathrm{v}_{0}$, which is the range? Ans.: 200 m
8. A ball is thrown vertically upwards with an initial speed of $20 \mathrm{~m} / \mathrm{s}$ from the roof of a building 50 m tall. The ball is pushed by the wind, with an horizontal acceleration of $2 \mathrm{~m} / \mathrm{s}^{2}$. Taking $g=10 \mathrm{~m} / \mathrm{s}^{2}$, calculate: a) Horizontal distance between the initial point and the point of impact. b) Maximum height. c) Intrinsic components of the acceleration when the ball reaches 60 m above ground the first time. Ans: a) $32,97 \mathrm{~m}$; b) 70 m ; c) $\mathrm{at}_{\mathrm{t}}=-9,02 \mathrm{~m} / \mathrm{s}^{2} ; \mathrm{a}_{\mathrm{n}}=4,32 \mathrm{~m} / \mathrm{s}^{2}$.
9. William Tell aims to the apple atop his son's head, at a distance $d$ and 50 cm below from shooting position. The initial speed of the arrow is $50 \mathrm{~m} / \mathrm{s}$, with an angle $30^{\circ}$. The wind produces an horizontal acceleration of 2 $\mathrm{m} / \mathrm{s}^{2}$, braking the arrow. Taking $g=10 \mathrm{~m} / \mathrm{s}^{2}$, calculate: a) Horizontal distance $d$ required for the arrow to go through the apple; b) Maximum height the arrow reaches, as measured from point of shooting. Ans: a) 192,4 m; b) $31,3 \mathrm{~m}$
10. See figure. A body is thrown from a point $P$ in the slope $\left(22^{\circ}\right)$ of a hill. The initial speed is $\mathrm{v}_{0}=150 \mathrm{~m} / \mathrm{s}$, forming an angle of $31^{\circ}$ with respect to the mountainside (see figure). Calculate the flight time, the distance from P when it hits the ground and the

